and the other conditions to remain the same. Develop into a series the expression $\left\{\left(1-\frac{c}{a\,x}\right)+\frac{c}{a\,x}\right\}^{t\,x}$, in which x represent the number of instants in a minute, thus:

$$\left\{\left(1-\frac{c}{a\,x}\right)+\frac{c}{a\,x}\right\}^{t\,x}=\left(1-\frac{c}{a\,x}\right)^{t\,x}+t\,x\left(1-\frac{c}{a\,x}\right)^{t\,x-1}\frac{c}{a\,x}$$
$$+\frac{t\,x(t\,x-1)}{2}\left(1-\frac{c}{a\,x}\right)^{t\,x-2}\frac{c^2}{a^2\,x^2}+\text{etc.}$$

If we now represent the parts of wine in each cask at the end of t minutes, beginning with the first, respectively by u, u', u'', u''' ets., then will

$u=\left(1-\frac{c}{a\,x}\right)^{t\,x}$, a minimum when $t=\infty$

$u'=t\,x\left(1-\frac{c}{a\,x}\right)^{t\,x-1}\frac{c}{a\,x}$, a maximum when $t=\frac{a}{c}$

$u''=\frac{t\,x(t\,x-1)}{2}\left(1-\frac{c}{a\,x}\right)^{t\,x-2}\frac{c^2}{a^2\,x^2}$, a maximum when $t=\frac{2\,a}{c}$

$u'''=\frac{t\,x\,(t\,x-1)\,(t\,x-2)}{2.\quad 3}\left(1-\frac{c}{a\,x}\right)^{t\,x-3}\frac{c^3}{a^3\,x^3}$ a maximum when $t=\frac{3\,a}{c}$

etc., etc., etc.

The values of u, u' u'', u''', etc., may readily be found by developing the above expressions by the binomial theorem and substituting $x=\infty$ or by applying the principles of logarithms as the student may prefer.

PROBLEMS.

25. By R. L. Seldon, Troy, N. Y.—Required the sides of an obtuse angled triangle the area of which is 14.048 acres, the obtuse angle $111^0 15'$, and one of the acute angles $11^o\,44'\,10''$.

26. By Wm. Hoover, South Bend, Ind.—Find θ from the equation $15\sin\theta+12\cos\theta=17.97240$.....(1).

27. By D. J. McAdam, Washington, Pa.—Four given equal spheres being placed in close contact with each other, it is required to find the volume of the space inclosed between them and the three triangular planes through each three centers.

29. By Prof. A. B. Evans, Lockport N. Y.—If $a, b, c, d, e, f, g, h, i, j, k$ be chords drawn from any point on the circumference of a circle to the eleven angles of an inscribed regular polygon of eleven sides; prove that $(a+k)(b+j)(c+i)(d+h)(e+g)=f^5$.....(1)